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VIBRATION OF COMPOSITE STRUCTURES.(U)

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by

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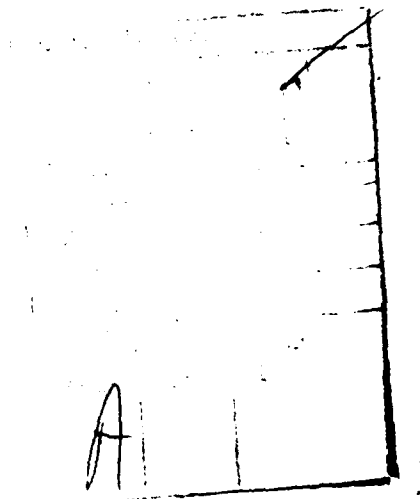
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## VIBRATION OF COMPOSITE STRUCTURES

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## 1. INTRODUCTION

The purpose of this paper is to review some of the recent contributions to the field of vibration of composite structures. The topics of fatigue and fracture, while quite important in the overall dynamic performance of composite structures, are not considered in this survey.

The survey treats these topics: dynamic stiffness and damping of composite materials, beams and curved bars and rings, flat panels, and cylindrically curved panels and shells. It concludes with some suggestions for future research.

## 2. DYNAMIC STIFFNESS AND DAMPING OF COMPOSITE MATERIALS

2.1 Stiffness of Unidirectional Fiber-Reinforced Material

It is well established that fiber-reinforced composite materials, by virtue of the geometry of the fibers acting as stiffeners, behave macroscopically like homogeneous but anisotropic materials. The class of elastic symmetry depends upon the geometric arrangement of the fibers (Table 1).

Table 1. Effect of Fiber Arrangement on Elastic Symmetry of Unidirectional Composites

<u>Arrangement of Fibers in Cross Section</u>	<u>Practical Examples</u>	<u>Class of Elastic Symmetry</u>
Very-small-diameter fibers packed statistically isotropically	Glass/resin, graphite/resin	Transversely isotropic (isotropic in fiber cross-sectional plane)
Hexagonally packed fibers	Same	Same
Fibers arranged in a single row or in a rectangular array	Boron/epoxy tape	Orthotropic with planes of sym- metry intersecting in the fiber direction and directions of rows and columns of fibers

Assuming linear stress-strain behavior, the generalized Hooke's law may be written in compliance matrix form, using contracted notation, as follows:

$$\{\epsilon_i\} = [S_{ij}]\{\sigma_j\} \quad (1)$$

where in the three-dimensional case, the strain matrix and stress matrix are

$$\begin{aligned} \{\epsilon_i\} &= (\epsilon_1, \dots, \epsilon_6)^T = (\epsilon_{11}, \epsilon_{22}, \epsilon_{33}, \gamma_{23}, \gamma_{31}, \gamma_{12})^T \\ \{\sigma_j\} &= (\sigma_1, \dots, \sigma_6)^T = (\sigma_{11}, \sigma_{22}, \sigma_{33}, \tau_{23}, \tau_{31}, \tau_{12})^T \end{aligned} \quad (2)$$

Here  $\epsilon_{ij}$  = normal strain,  $\gamma_{ij}$  = engineering shear strain,  $\sigma_{ij}$  = normal stress, and  $\tau_{ij}$  = shear stress.

For the orthotropic case, the compliance matrix takes the form

$$[S_{ij}] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \quad (3)$$

For the case of transverse isotropy, with the 23 plane as the plane of isotropy, array (3) takes the following special form:  $S_{13} = S_{12}$ ,  $S_{33} = S_{22}$ ,  $S_{44} = 2(S_{11} - S_{12})$  and  $S_{55} = S_{66}$ .

Equation (1) can be inverted to obtain the 3-D stiffness form as follows:

$$\{\sigma_j\} = [C_{ij}]\{\epsilon_j\} \quad (4)$$

Here, the  $C_{ij}$  are the three-dimensional Cauchy stiffness coefficients. Although equation (1) is applicable to one-, two-, or three-dimensional stress systems, equation (4) is applicable only in the case of 3-D stress systems. In the case of a relatively thin member, it is often acceptable to neglect the effect of thickness normal stress ( $\sigma_3$ ), then equation (1) is inverted to obtain the following expression

$$\{\sigma_i\} = [Q_{ij}]\{\epsilon_j\} \quad (5)$$

where the  $Q_{ij}$  are the plane-stress reduced stiffnesses which are related to the three-dimensional Cauchy stiffnesses as follows:

$$Q_{ij} = C_{ij} - (C_{i3}C_{j3}/C_{33}) \quad (6)$$

All of the above elastic coefficients ( $S_{ij}$ ,  $C_{ij}$ , and  $Q_{ij}$ ) can be related to the so-called engineering coefficients, Young's moduli  $E_i$ , Poisson's ratios  $\nu_{ij}$ , and shear moduli  $G_{ij}$ ; see, for example, [1,2,3].

It is noted that both the compliance  $[S_{ij}]$  and stiffness ( $[C_{ij}]$  or  $[Q_{ij}]$ ) matrices are symmetric as a consequence of conservation of strain energy, as was first shown very elegantly by George Green in the early 1800's. This reduces the number of independent elastic coefficients in every case. It is emphasized that in writing all of the preceding equations the stresses, strains, and elastic coefficients are all related to material-symmetry axes, i.e., axes formed by the intersection of mutually perpendicular planes of symmetry. In a unidirectional-fiber-reinforced composite, one axis is obviously the fiber direction.

If one wishes to express the stresses, strains, or elastic coefficients with respect to an orthogonal coordinate system oriented with its  $z$  axis in line with direction  $x_3$ , with its  $x, y$  axes oriented at an angle  $\theta$  with respect to the  $x_1, x_2$  axes, standard tensor relations can be used to obtain appropriate transformation equations [1,2,3]. Then

$$\{\bar{\epsilon}_i\} = [\bar{S}_{ij}]\{\bar{\sigma}_j\} \quad (7)$$

The bar denotes that the quantity is related to the  $(x, y, z)$  coordinate system.

It is noted that in the array of equation (7), certain of the coefficients ( $C_{16}$ ,  $C_{26}$ , and  $C_{45}$ ) that were identically zero in the orthotropic case, equation (3), are no longer zero. Thus,

$$[\bar{S}_{ij}] = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{13} & 0 & 0 & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{23} & 0 & 0 & \bar{S}_{26} \\ \bar{S}_{13} & \bar{S}_{23} & \bar{S}_{33} & 0 & 0 & \bar{S}_{36} \\ 0 & 0 & 0 & \bar{S}_{44} & \bar{S}_{45} & 0 \\ 0 & 0 & 0 & \bar{S}_{45} & \bar{S}_{55} & 0 \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{36} & 0 & 0 & \bar{S}_{66} \end{bmatrix} \quad (8)$$

The number of non-zero coefficients has increased from 12 to 20 and the number of independent ones has increased from 9 to 13. The elastic symmetry class represented by equation (8) is called monoclinic. In other words, a material that is orthotropic with respect to symmetry axes 1,2,3 behaves like a monoclinic material with respect to axes x,y,z as defined above.

Table 2 summarizes the various symmetry classes of importance in fiber-reinforced composite materials.

Table 2. Summary of Selected Elastic Symmetry Classes

<u>Symmetry Class</u>	<u>Total No. of Non-Zero Elastic Coefficients</u>	<u>No. of Independent Elastic Coefficients</u>
Three-Dimensional-Stress Cases:		
General anisotropic	36	21
Monoclinic	20	13
Orthotropic	12	9
Transversely isotropic	12	5
Isotropic	12	2
Plane-Stress Cases:		
General anisotropic = monoclinic (symm. about $x_3=0$ )	9	6
Orthotropic (= transversely isotropic in $x_2x_3$ plane)	5	4
Transversely isotropic in $x_1x_2$ plane (= isotropic)	5	2

## 2.2 Stiffness of Laminates

To obtain certain desired combinations of properties, such as stiffnesses and strengths, it is usually necessary to arrange individual layers (sometimes called laminae) of fiber-reinforced composite materials to form a multi-layer laminate. It is conceptually possible to treat structural elements constructed of laminates by applying the relations of three-dimensional elastodynamics to each individual layer in conjunction with appropriate continuity and boundary conditions. However, the resulting analysis is conceptually so complicated that it is not practical to carry out except in a few very special cases. To avoid this complexity, it is the usual engineering practice to make certain hypotheses regarding the kinematics of deformation and then to work with stiffnesses which are integrated through the thickness.

The simplest kinematic assumption is the Bernoulli-Euler one for slender beams (or the Kirchhoff-Love one for thin plates or shells): plane sections remain plane and normal to the deflected middle surface during deformation, and undergo no thickness stretching. In view of the low shear modulus of most composite materials relative to their in-plane elastic moduli, it is more accurate to use the Bresse-Timoshenko hypothesis [4,5] for beams (or the Reissner-Mindlin one [6,7] for plates and shells). In this hypothesis, it is assumed that plane sections remain plane but not necessarily normal to the deflected middle surface and suffer no thickness stretching. Thus, the additional deflection due to thickness-shear deformation is accounted for in an approximate sense. Mathematically

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\beta_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\beta_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad (9)$$

Here  $u, v$  are displacements in the  $x, y$  directions;  $w$  is the normal deflection;  $u_0, v_0, w_0$  are the middle-surface displacements in the  $x, y, z$  directions;  $(\cdot)_x \equiv \partial(\cdot)/\partial x$ ;  $\beta_x$  and  $\beta_y$  are the bending slopes and differ from the total slopes by the respective shear strains. Using equations (9) in the equations of linear elasticity theory, one gets

$$\varepsilon_i(x,y,z) = \varepsilon_i^0(x,y) + z\kappa_i(x,y) \quad (i=1,2,6) \quad (10)$$

Here  $\varepsilon_i^0$  are the mid-plane stretching and shearing strains and  $\kappa_i$  are the bending and twisting curvatures.

Now the stress resultants and stress couples are related to the corresponding stress components as follows for a flat laminate (plate):

$$\begin{aligned} N_i &= \int_{-h/2}^{h/2} \bar{\sigma}_i dz \quad (i=1,2,6) ; \quad Q_i = \int_{-h/2}^{h/2} \bar{\tau}_i dz \quad (i=4,5) \\ M_i &= \int_{-h/2}^{h/2} z \bar{\sigma}_i dz \quad (i=1,2,6) \end{aligned} \quad (11)$$

Here  $N_i$  are in-plane forces per unit width,  $Q_i$  are thickness shear forces per unit width,  $M_i$  are bending or twisting moments per unit width, and  $h$  is the total laminate thickness.

Substituting equations (5) and (10) in equations (11) and performing the integrations, one obtains the following laminated plate constitutive relations:

$$\begin{aligned} \begin{Bmatrix} N_i \\ M_i \end{Bmatrix} &= \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j^0 \\ \kappa_j \end{Bmatrix} \quad (i,j=1,2,6) \\ \{Q_i\} &= [A_{ij}]\{\varepsilon_j\} \quad (i,j=4,5) \end{aligned} \quad (12)$$

Here the  $A_{ij}$  ( $i,j=1,2,6$ ) are in-plane (stretching and shearing) stiffnesses,  $A_{ij}$  ( $i,j=4,5$ ) are thickness-shear stiffnesses,  $B_{ij}$  are in-plane/out-of-plane coupling stiffnesses, and  $D_{ij}$  are out-of-plane (bending and twisting) stiffnesses given by:

$$\begin{aligned}
 A_{ij} &= \int \bar{Q}_{ij} dz = \sum_{l=1}^n (z_l - z_{l-1}) \bar{Q}_{ij}^{(l)} \\
 B_{ij} &= \int z \bar{Q}_{ij} dz = \frac{1}{2} \sum_{l=1}^n (z_l^2 - z_{l-1}^2) \bar{Q}_{ij}^{(l)} \quad (i,j=1,2,6) \\
 D_{ij} &= \int z^2 \bar{Q}_{ij} dz = \frac{1}{3} \sum_{l=1}^n (z_l^3 - z_{l-1}^3) \bar{Q}_{ij}^{(l)} \\
 A_{ij} &= k_i k_j \int \bar{Q}_{ij} dz = k_i k_j \sum_{l=1}^n (z_l - z_{l-1}) \bar{Q}_{ij}^{(l)} \quad (i,j=4,5)
 \end{aligned} \tag{13}$$

Here index  $l$  refers to the  $l$ -th layer,  $n \equiv$  total number of layers,  $z_{l-1}$  and  $z_l$  are  $z$ -coordinate positions of the top and bottom surfaces of the typical layer  $l$ , and the  $k_i$  are the shear-correction coefficients to account for the nonlinear distribution of the thickness shear strains through the total thickness.

The basic difference between the theories of Refs. [8] and [9] is that the former defined the plate stiffnesses ( $A_{ij}$ , etc.) on the basis of the three-dimensional material stiffness coefficients ( $\bar{Q}_{ij}$ ) while the latter defined them in terms of the  $\bar{Q}_{ij}$  as in equations (13). Wang and Chou [10] showed that the latter approach gives more accurate results even for relatively thick plates.

There is a nearly unlimited number of combinations of layer orientations possible in a laminate. Table 3 lists commonly used lamination arrangements consisting of multiple plies all having the same thickness and of identical composite material (except for orientation).

Table 3. Characteristics of Some Commonly Used Lamination Arrangements

Laminate Name	Description	Stiffness Characteristics	Ref.
Aligned parallel-ply	All layers at $0^\circ$	$A_{16}=A_{26}=D_{16}=D_{26}=\text{all } B_{ij}=0$	[11]
Off-axis parallel-ply	All layers at $\theta$	All $B_{ij}=0$	[11]
Symmetric cross-ply	Alternating $0^\circ$ & $90^\circ$ layers; $n=\text{odd}$	$A_{16}=A_{26}=D_{16}=D_{26}=\text{all } B_{ij}=0$	[3,11,12]
Antisymmetric cross-ply	Alternating $0^\circ$ & $90^\circ$ layers; $n=\text{even}$	$A_{16}=A_{26}=D_{16}=D_{26}=0$ ; $B_{11}=-B_{22}$ ; remainder of $B_{ij}=0$	[3,11,12]
Symmetric regular angle-ply	Alternating $+\theta$ & $-\theta$ layers; $n=\text{odd}$	All $B_{ij}=0$ ; $A_{16}, A_{26}, D_{16}, D_{26}$ decrease as $n$ is increased	[3,11,12]
Antisymmetric regular angle-ply	Alternating $+\theta$ & $-\theta$ layers; $n=\text{even}$	$A_{16}=A_{26}=B_{11}=B_{22}=B_{66}=D_{16}=D_{26}=0$	[3,11,12]
Symmetric balanced angle-ply	Center 2 plies at $\theta$ ; remainder alternating $+\theta$ & $-\theta$ ; $n=\text{even}$	$A_{16}=A_{26}=\text{all } B_{ij}=0$ ; $D_{16}$ and $D_{26}$ decrease as $n$ is increased	[13]
Quasi-isotropic (isotropic $A_{ij}$ only)	One or more sets of layers at $0, \theta_1, 2\theta_1, 3\theta_1, \dots, (N-1)\theta_1$ , where $\theta_1=180^\circ/N$ , $N \equiv$ no. of different orientations (integer $\geq 3$ )	$A_{11}=A_{22}=A$ , $A_{12}=\nu A$ , $A_{66}=(1-\nu)(A/2)$ , $A_{16}=A_{26}=0$	[3,11]



### 2.3 Stiffness of Composites with Different Behavior in Tension and Compression

Analysis of materials having different stress-strain relations in tension and compression dates back to Saint-Venant (1864); see [14]. Examples of materials which have been found to demonstrate this behavior include various cords and fabric embedded in rubber, sintered metal, and certain biological materials [14]. Modern treatment of the topic dates from the 1965 work of Ambartsumyan [15], in which he instituted the concept of a bimodulus model for isotropic materials with different properties in tension and compression. This concept was later generalized to anisotropic material by the present investigator [16, 17] using a fiber-governed symmetric elastic model which was found to agree well with experimental results for cord/rubber.

In the fiber-governed symmetric elastic model [16,17], it is assumed that there is a complete array of stiffnesses which govern when the fiber-direction strain ( $\epsilon_f$ ) is tensile and another complete array when it is compressive. That is, for a thin laminate, the plane-stress reduced stiffness matrix is

$$[Q_{ijk}] = \begin{cases} [Q_{ij1}] & \text{if } \epsilon_f \geq 0 \\ [Q_{ij2}] & \text{if } \epsilon_f < 0 \end{cases} \quad (14)$$

Here the index  $k$  refers to either the tensile ( $k=1$ ) or compressive ( $k=2$ ) set of stiffnesses.

For a beam, plate, or shell undergoing bending, the locus of points at which  $\epsilon_f=0$  is called the neutral surface. In an ordinary (not bimodulus) material, the neutral-surface location is not especially important. However, in a bimodulus material, its location is crucial, and in general it does not coincide with the middle surface for such materials. Thus, even a single layer of bimodulus material exhibits bending-stretching coupling (analogous to an unsymmetrically laminated laminate), except when the magnitude of the middle-surface strain in the fiber direction exceeds the bending strain. In the latter exception, a bimodulus material acts no differently than an ordinary one.

Depending upon the structural configuration, boundary conditions, and loading, even in a single-layer plate, the neutral surface in general is not a horizontal plane. Since the layer properties ( $Q_{ijk}$ ) depend upon the sign of  $\epsilon_f$  (and thus upon the neutral-surface position) and the plate stiffnesses ( $A_{ij}$ ,  $B_{ij}$ ,  $D_{ij}$ ) upon the  $Q_{ijk}$  as in equations (13), it is clear that in general a bimodulus plate is nonhomogeneous in its plane, i.e., the plate stiffnesses depend upon position ( $x,y$ ) in the plane [18]. However, unlike a plate with tapering thickness, here the plate stiffnesses are not even known *a priori*. This situation is reminiscent of the elastoplastic bending problem, for which the elastoplastic boundary is not known *a priori*.

### 2.4 Damping of Filamentary Composites and Laminates

Although there are enumerable mathematical models which have been proposed to describe internal-friction in materials [19], by far the most widely used is the Kimball-Lovell model [20]. This model assumed that the energy dissipated is proportional to the square of the amplitude but independent of frequency. In the case of sinusoidal excitation, it is convenient to apply this model by replacing the elastic stiffnesses ( $Q_{ij}$ ) of a perfectly elastic material by the complex stiffnesses ( $\bar{Q}_{ij}$ ) defined by

$$\bar{Q}_{ij} = A_{ij}^R + iQ_{ij}^I \quad (15)$$

Here  $i \equiv \sqrt{-1}$ ,  $Q_{ij}^R \equiv$  real (storage) component of stiffness, and  $Q_{ij}^I \equiv$  imaginary (loss) component. Although, as discussed in [21], there are considerable difficulties in applying the complex-stiffness approach to nonsinusoidally excited systems, no alternative proposed to date appears to offer much promise, with the possible exception of the fractional-derivative approach originated by Caputo and recently applied to isotropic adhesive material by Bagley and Torvik [22]. Preliminary attempts to apply this model to boron-epoxy have not been successful.

The pioneering work in analysis of damping in fiber-reinforced composite materials was due to Hashin [23], using the elastic-viscoelastic correspondence principle. Unfortunately, this approach requires explicit expressions for the elastic stiffnesses, and such expressions are generally not sufficiently accurate for transverse and shear stiffnesses ( $Q_{22}$  and  $Q_{66}$ ). These limitations were removed in [24], a complete characterization of boron/epoxy.

A variety of experimental techniques have been used to experimentally characterize the dynamic stiffness and damping of composite materials. Reference [25] discussed the free vibration, pulse propagation, and forced vibratory response methods. Selected recent sources of experimental data for various filamentary composites are listed in Table 4.

Table 4. Selected Sources of Experimental Data on Damping of Fiber-Reinforced Composite Materials

<u>Fiber Material</u>	<u>Fiber Form</u>	<u>Matrix Material</u>	<u>Ref.</u>
Glass	Continuous	Epoxy	26,27
Boron	Continuous	Epoxy	28,29
Graphite	Continuous	Epoxy	28,30
Glass	Short	Polyester	31
Aramid	Woven cloth	Polyester	32

### 3. BEAMS, CURVED BARS, AND RINGS

Reference [33] surveyed the static behavior of such structural elements and discussed 25 pertinent references through 1973. Here emphasis is placed on the structural dynamics aspects.

#### 3.1 Straight Beams of Anisotropic Material

For analyzing the lower flexural modes of straight beams made of orthotropic material (aligned fiber-reinforced composite material), the dynamic version of the simple Bernoulli-Euler type theory is adequate. Of course, the appropriate Young's modulus (the longitudinal one) must be used in defining the flexural rigidity.

Since composite materials typically have much lower ratios of shear modulus to Young's modulus than homogeneous isotropic materials, they usually exhibit more transverse shearing action than isotropic materials. In the aligned (orthotropic) case, one may use the equations of Bresse-Timoshenko beam theory [4,5], provided that one uses the appropriate elastic moduli: longitudinal Young's modulus for  $E$  and the appropriate (thickness shear or in-plane shear) shear modulus for  $G$ . Apparently this approach was used for orthotropic beams first by Nowinski [34] and later by others [35-36].

To improve the dynamic analysis of orthotropic beams still further, it is, of course, necessary to use a three-dimensional elastodynamic approach. Such a

procedure was used, for example, by Ohnabe and Nowinski [37], who extended the Pochhammer-Chree equations of isotropic elastodynamics to the cylindrically orthotropic case.

For beams made of off-axis or nonaligned fiber-reinforced material (i.e., general anisotropic material), there is an elastic coupling between the flexural and twisting actions. This was analyzed by Lekhnitskii [2], sections 27 and 28, for the static case. Apparently the first dynamic analysis including this coupling was Abarcar and Cunniff's 1972 analysis [38], who combined Bresse-Timoshenko beam theory with Lekhnitskii's flexural-torsional coupling theory. They formulated the problem as one in Myklestad's transfer-matrix theory [39]. Further experimental and transfer-matrix analyses were carried out by Ritchie et al. [40-41]. Later Miller and Adams [42] obtained a purely analytical solution for a theory based on Bernoulli-Euler beam theory rather than the Bresse-Timoshenko one.

Recently Teoh and Huang [43] presented a closed-form solution for free vibrations of Bresse-Timoshenko-Lekhnitskii beams and Teh and Huang [44] analyzed the same problem using finite elements based on an energy formulation. Most recently Wallace and Bert [45] extended the transfer-matrix approach to include Kimball-Lovell material damping. At a considerable saving in computational time, they were able to predict all but the pure "plate modes" and associated frequencies and damping factors for a wide beam previously investigated experimentally [46] and analytically [47].

Mindlin [48] presented a three-dimensional dynamic analysis of the low-frequency modes of anisotropic elastic bars of arbitrary cross section and applied it to bars with elliptic, triangular, and rectangular cross sections.

### 3.2 Laminated and Sandwich Straight Beams

A laminated beam is one having two or more layers, with all layers of the same thickness but of different materials (or fiber orientations). In contrast a sandwich beam is one having one (or more) thick, flexible core and two (or more) thin, stiff facings. A simple sandwich has one core between two facings, while a multicore sandwich ("club sandwich") has more than one core sandwiched between more than two facings [49]. Actually a multilayer beam with constrained viscoelastic layers can be considered to be a multicore sandwich.

Although laminated beams are simpler in geometry than laminated plates, relatively few vibrational analyses of this configuration have appeared in the recent literature, exemplified by [50-54].

Since there is a voluminous literature on the linear analysis of free and sinusoidally forced vibration of simple sandwich beams (much of the early work was reviewed in [55]), attention here is focused on the literature of the past eight years with particular attention to various complexities. The effects of concentrated masses were investigated in [56-57], while Ref. [58] considered the effect of local shearing prevention, such as that occurring due to the presence of rivets near the ends. The effects of various loadings have been investigated; these included impact [59], moving force systems [60], and flutter [61]. Hyer et al. [62] investigated the effect of geometric nonlinearity, both analytically and experimentally.

The effects of a variety of configuration parameters on sandwich-beam vibration have been studied. Sandman [63] considered the effects of segmenting, i.e., discrete changes in stiffness and mass along the length of the beam, while D.K. Rao and Stuhler [64] investigated the effect of a smoothly tapering cross section. The presence of pre-twist, as in an aerodynamic surface such as a compressor blade, was shown by D.K. Rao [65] to result in drastic reduction of

the effective loss factor for the first flexural mode. An unsymmetric sandwich is one which does not have mid-plane symmetry. Vibrational analyses of unsymmetric sandwich beams having different top and bottom facings were presented by Yan and Dowell [66], Y.V.K.S. Rao and Nakra [67], and by D.K. Rao [68-69]. Reference [69] also included the effect of multiple cores. Y.V.K.S. Rao [70] analyzed an unsymmetric sandwich beam that he called a dual-core sandwich configuration; it consisted of two different core materials (back-to-back) sandwiched between only two facings.

### 3.3 Curved Beams and Rings

The governing equations of motion for curved beams and rings are identical; the difference comes in via the boundary conditions. A curved beam or arch has specified boundary conditions at each end, while a ring must have a solution which is circumferentially continuous, i.e., periodic in the circumferential angular position. Although there have been many vibrational analyses of arches, curved beams, and rings, apparently the only one strictly applicable to a composite ring is [71]. It treated free vibration of an orthotropic-material thick ring subjected to steady centrifugal loading, supported by continuous radial elastic support, and included transverse shear deformation. The application was a hoop-wound rim-type flywheel for automotive energy-storage applications.

There have been a few dynamic analyses of curved sandwich beams and rings. Ahmed [72] performed a finite-element analysis of a curved beam. References [73-75] analyzed the steady-state dynamic response of damped sandwich rings, while Sagartz [76-77] considered the transient response.

## 4. FLAT PANELS (PLATES)

Since publication of Leissa's comprehensive monograph [78] on free vibration of plates in general, there have been a number of review articles [33,79-81] which surveyed the vibration of composite-material and sandwich plates. Thus, in the interests of brevity, only a few very recent developments not discussed in the above references are considered here.

### 4.1 Plates of Anisotropic Material

Linear analyses included the following: for orthotropic rectangular plates, Wilson [82] considered the response to moving loads, while for free vibration of anisotropic rectangular plates subjected to in-plane forces, Laura and Luisoni [83] presented a Ritz-type solution with a polynomial approximating function. Reference [84] considered moderately thick rectangular plates of bi-modulus orthotropic material. Reference [85] discussed the application of a combination of the finite-strip method with the deflection-contour method and obtained reasonably good results for an orthotropic square plate.

Linear analyses of cylindrically orthotropic plates included a power-series solution of a spinning annular plate with simply supported reinforcing beams at both edges [86], finite-element and time-averaged holographic investigations of annular plates [87], and a spline-function Ritz analysis of plates having a sector planform [88].

Nonlinear analyses with geometric nonlinearity included two papers on orthotropic, moderately thick, rectangular plates [89-90]. Also there was a recent nonlinear analysis of anisotropic skew (parallelogram) plates [91].

### 4.2 Laminated Plates of Anisotropic Material

There have been relatively few analyses of laminated plates, since the

latest survey [81]. Rao and Singh [92] presented an optimal design synthesis procedure to design rectangular plates of minimum weight subject to constraints on natural frequencies. A finite-element analysis including thickness-shear flexibility was developed by Reddy [93] and compared with various results in the literature. Crawley [94] reported extensive experimental results for cantilever rectangular plates of graphite/epoxy and graphite/epoxy-aluminum and compared the results with moderately thick finite-element predictions. Chatterjee and Kulkarni [95] analyzed flutter of moderately thick rectangular panels of damped, laminated composite materials.

Elishakoff and Stavsky [96] analyzed vibration of laminated cylindrically orthotropic circular annular plates.

## 5. CYLINDRICALLY CURVED PANELS AND COMPLETE CYLINDERS

This area has been surveyed in Leissa's comprehensive monograph [97] on free vibration of shells in general and in a number of review articles [33, 98, 99] on vibration of composite-material shells. Here, in the interests of brevity, only cylindrical shells, either cylindrically curved panels or circumferentially complete cylinders, are reviewed. Emphasis is placed on work carried out from 1973 to the present.

It should be recalled that for plates there are primarily two classes of analyses (linear and nonlinear), three kinematic situations (3-D, moderately thick, or thin), and two material-geometry cases (single-layer/symmetrically laminated, and arbitrarily laminated). In the case of shells, there is a much greater variety in the kinematic situations. The hierarchy of shell theories, from most accurate to least accurate (which may be either with or without thickness-shear and thickness-normal action)[100] is:

- 1) Exact theory, like the Langhaar-Boresi theory
- 2) Second-approximation theories, like Love's second-approximation theory, Flugge's and Novozhilov's
- 3) First-approximation theories, such as Love's first approximation theory, and the increasingly popular Sanders theory
- 4) Morley's shallow-shell theory
- 5) Donnell's very-shallow-shell theory

The selection of an appropriate theory depends upon the method of solution. For example, if a whole-shell analytical method is used for a complete shell, the Donnell and Morley theories may not be sufficiently accurate for predicting the frequencies of the lower circumferential modes. On the other hand, if the shell is discretized into many small regions, each of which is individually very shallow, then the Morley or Donnell theory may be entirely adequate and the complexity of the more elaborate theories may not be justified.

### 5.1 Shells of Anisotropic Material

Since this theory is relatively simple, especially for first-approximation or simpler theory and orthotropic material, recent emphasis has been on complicating effects.

Babu and Reddy [101] presented one of the relatively few analyses of cylindrical panels. Fortier [102] considered the effect of external pressure on cylindrical panel vibration. Dym [103] investigated the effect of this same loading on complete cylinders, while Penzes and Kraus [104] also added the effects of torsion, axial force, and rotation about the axis (centrifugal loading). Kuptsov [105] considered the effect of centrifugal loading due to rotation about a shell diameter at one end of the shell. The effects of internal

irrotational flow [106-107], external flow [108], and internal swirling flow [109-110] on orthotropic pipelines have been analyzed, as have the effects of random boundary-layer internal-pressure fields [111].

In one of the relatively few analyses of nonlinear vibration of a shell of nonlinear anisotropic material, Shahinpoor [112] analyzed a thin tube of Ericksen-Rivlin material. The large-amplitude (geometrically nonlinear) vibrations of an orthotropic cylindrical shell with axially tapering thickness was recently investigated by Ramachandran [113].

Concerning moderately thick orthotropic shells, the work of Warburton and Soni [114] on harmonically excited vibration, including material damping, is one of the most extensive. Mangrum and Burns [115] considered the effect of a discontinuous pressure loading moving at constant axial velocity. Jain [116] analyzed the vibration of a vertical cylinder partially filled with fluid, while Shvets and Marchuk [117] considered a pipe with an internally flowing fluid.

All of the work just mentioned was limited to orthotropic material. One of a few vibrational investigations of anisotropic (monoclinic) cylindrical shells of moderately thick walls was recently reported in [118]. The analysis was formulated in terms of a higher-order shell theory which retained some of the exact kinematic features of the Langhaar-Boresi shell theory and yet included thickness-shear deformation as in Reissner-Naghdi theory. The predicted natural frequencies were found to agree reasonably well with experimentally determined ones for a cylinder of unidirectional material oriented at 30 degrees to the axis of the cylinder.

## 5.2 Laminated and Sandwich Shells

The effects of several different boundary conditions on free vibrations of laminated, circular cylindrical shells were investigated analytically by Abhat and Wilcox [119] and Fortier and Rossettos [120]. Reference [119] is particularly interesting in that it introduced a new, improved method of reducing the general eighth-order shell frequency determinant to a fourth-order one. The results obtained using this method are considerably more accurate than those obtained by Yu's reduction method [121]. In [119] the new method was applied to cross-ply shells with either clamped or freely supported ends. In [120], four different edge conditions were considered for both cross-ply and angle-ply laminates. In [122], four different boundary conditions were investigated experimentally.

In one of the relatively scarce analyses of noncircular laminated cylindrical shells, Noor [123] applied the so-called multi-local method, which is a variation of the Hermitian difference technique. Optimization problems relating to vibration of laminated anisotropic circular cylindrical shells were investigated in [124-125].

A number of investigators considered the effects of various kinds of external loadings on free vibration of laminated cylindrical shells. References [126] and [127] considered external pressure; Padovan [128] included pressure, axial load, torque, and centrifugal and Coriolis forces.

Berger [129] analyzed the vibrations of an infinitely long, layered orthotropic cylindrical shell in an acoustic medium. Muggeridge and Buckley [130] carried out analytical and experimental investigations on symmetric balanced angle-ply cylinders in a fluid.

There have been a number of analyses of moderately thick circular cylindrical shells, i.e., those with thickness-shear flexibility explicitly included. Fortier and Rossettos [131] considered shallow cylindrically curved panels of

cross-ply lamination. Sinha and A.K. Rath [132] considered the same geometry subject to free vibration and buckling. B.K. Rath and Das [133] analyzed cross-ply laminated complete cylindrical shells by three shell theories: a refined moderately thick-shell theory, "classical" thin-shell theory (Love's first approximation), and Donnell's thin-shell theory.

Sun and Whitney [134] presented a theory for moderately thick laminated cylindrical shells. This theory was applied by C.T. Sun and P.W. Sun [135-136] to several cases of suddenly applied loading, using classical separation of variables and the Mindlin-Goodman technique.

Various applications of the finite-element technique have been made to moderately thick, laminated, circular cylindrical shells. Shivakumar and Murty [137] developed a ring-type element with sixteen degrees of freedom. Crawley [94] compared FEM and experimental results. Bradford and Dong [138] devised a refined element, in which a number of elements through the thickness comprise a laminate, and applied it to shells under initial stress.

There have been relatively few three-dimensional elastodynamic analyses of laminated cylinders. Two examples of such analyses are those of Srinivas [139] and Muiholland and Gupta [140].

Recently the activity in vibration of sandwich shells has diminished in comparison to its vitality in the 1960's. However, one should mention the theoretical and experimental investigations by Harari and Sandman [141-142] on sandwich shells with graphite/epoxy facings and the nonlinear parametric vibration analysis of Popov et al. [143].

## 6. SUGGESTIONS FOR FUTURE RESEARCH

The author believes that the following needs are most pressing in the areas covered by this survey.

1. More realistic mathematical modeling of material behavior is urgently needed. This includes such effects as material damping, stress-strain non-linearity in shear, different behavior in tension and compression, and effects of temperature, humidity, and material damage on the stiffness properties. Experimental verification of the improved models is also of importance.

2. Means for more realistic incorporation of practically important localized discontinuities should be developed. These factors include local doublers and edge reinforcements, access ports and other cutouts, and attached localized masses.

3. A comprehensive and comparative assessment of the numerous laminated shell theories presently available is sorely needed. The goal should be to determine the simplest theory necessary for practical engineering calculation of the structural dynamic response of composite shell structures.

4. Development of reasonably general design data to guide the structural designer in application of composites in dynamically loaded structures is most urgently needed.

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# PREVIOUS REPORTS ON THIS CONTRACT

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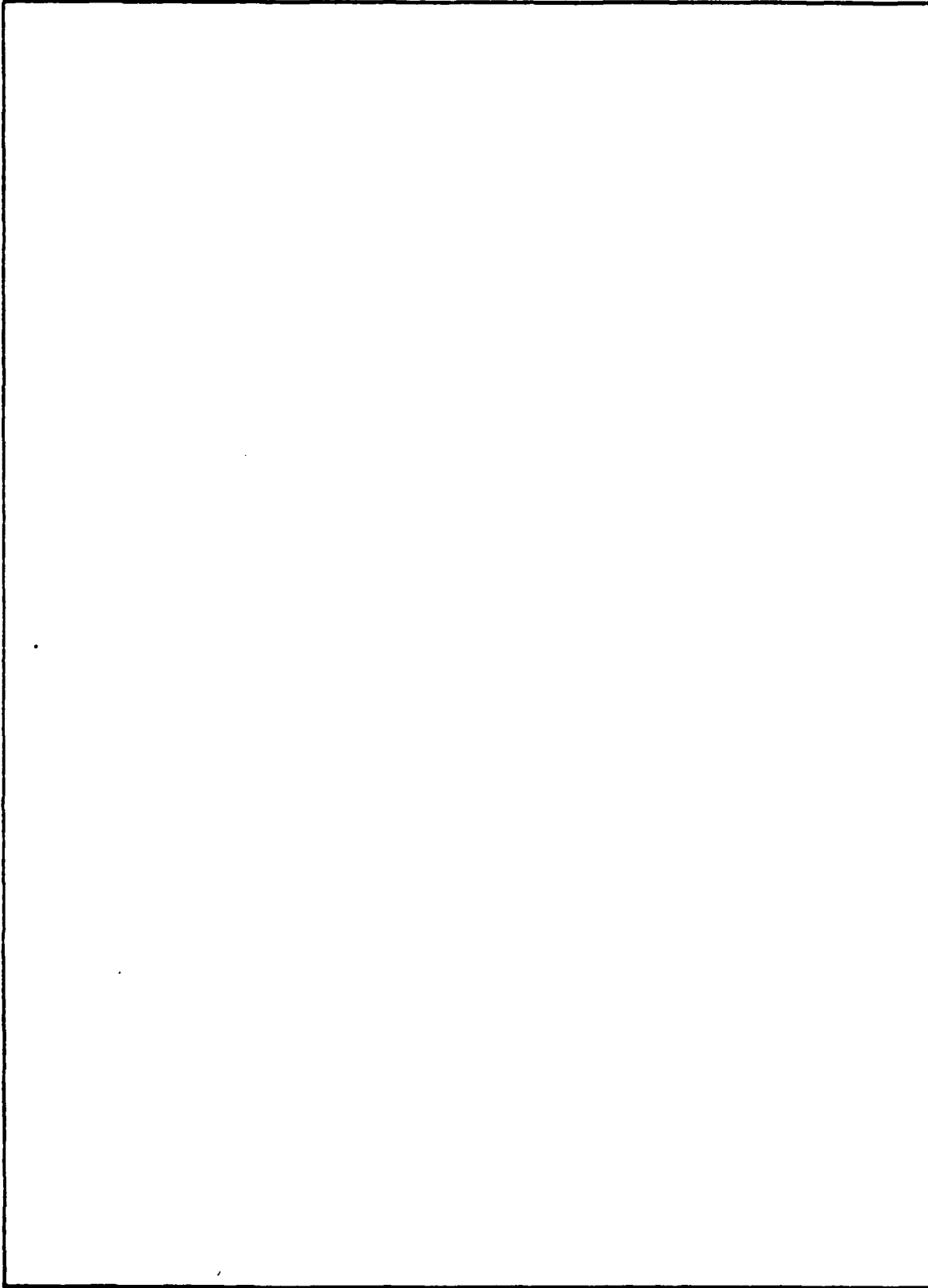
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18. SUPPLEMENTARY NOTES This is an invited lecture which is to appear in the Proceedings of the International Conference on Recent Advances in Structural Dynamics to be held July 7-11, 1980 at the Institute of Sound and Vibration Research, University of Southampton, Southampton, England.		
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The survey treats these topics: dynamic stiffness and damping of composite materials, beams and curved bars and rings, flat panels, and cylindrically curved panels and shells. It concludes with some suggestions for future research.		

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